Sheaves In Geometry And Logic A First Introduction To Topos Theory

PHELPS BROOKLYN

An Introduction to Noncommutative Geometry Springer Science & Business Media
This text exposes the basic features of cohomology of sheaves and its applications. The general theory of sheaves is very limited and no essential result is obtainable without turn ing to particular classes of topological spaces. The most satis factory general class is that of locally compact spaces and it is the study of such spaces which occupies the central part of this text. The fundamental concepts in the study of locally compact spaces is cohomology with compact support and a particular class of sheaves,the so-called soft sheaves. This class plays a double role as the basic vehicle for the internal theory and is the key to applications in analysis. The basic example of a soft sheaf is the sheaf of smooth functions on ~n or more generally on any smooth manifold. A rather large effort has been made to demon strate the relevance of sheaf theory in even the most elementary analysis. This process has been reversed in order to base the fundamental calculations in sheaf theory on elementary analysis.
Projective Geometry Springer Science & Business Media
Topos Theory is an important branch of mathematical logic of interest to theoretical computer scientists, logicians and philosophers who study the foundations of mathematics, and to those working in differential geometry and continuum physics. This compendium contains material that was previously available only in specialist journals. This is likely to become the standard reference work for all those interested in the subject.
Logic and Structure MIT Press
In presenting this treatment of homological algebra, it is a pleasure to acknowledge the help and encouragement which I have had from all sides. Homological algebra arose from many sources in algebra and topology. Decisive examples came from the study of group extensions and their factor sets, a subject I learned in joint work with OTTO SCHIL LING. A further development of homological ideas, with a view to their topological applications, came in my long collaboration with SAMUEL EILENBERG; to both collaborators, especial thanks. For many years the Air Force Office of Scientific Research supported my research projects on various subjects now summarized here; it is a pleasure to acknowledge their lively understanding of basic science. Both REINHOLD BAER and JOSEF SCHMID read and commented on my entire manuscript; their advice has led to many improvements. ANDERS KOCK and JACQUES RIGUET have read the entire galley proof and caught many slips and obscurities. Among the others whose sug gestions have served me well, I note FRANK ADAMS, LOUIS AUSLANDER, WILFRED COCKCROFT, ALBRECHT DOLD, GEOFFREY HORROCKS, FRIED RICH KASCH, JOHANN LEICHT, ARUNAS LIULEVICIUS, JOHN MOORE, DIE TER PUPPE, JOSEPH YAO, and a number of my current students at the University of Chicago - not to m~ntion the auditors of my lectures at Chicago, Heidelberg, Bonn, Frankfurt, and Aarhus. My wife, DOROTHY, has cheerfully typed more versions of more chapters than she would like to count. Messrs.
Categories and Sheaves Springer Science & Business Media
This book develops the theory of infinite-dimensional categories by studying the universe, or ∞-cosmos, in which they live.
Sheaf Theory Springer Science & Business Media
This is the first book on category theory for a broad philosophical readership. There is no other discussion of category theory comparable in its scope. It is designed to show the interest and significant of category theory for philosophers working in a range of areas, including mathematics, proof theory, computer science, ontology, physics, biology, cognition, mathematical modelling, the structure of scientific theories, and the structure of the world. Moreover, it does this in a way that is accessible to non specialists. Each chapter is written by either a category-theorist or a philosopher working in one of the represented fields, in a way that builds on the concepts already familiar to philosophers working in these areas. The book is split into two halves. The 'pure' chapters focus on the use of category theory for mathematical, foundational, and logical purposes, while the 'applied' chapters consider the use of category theory for representational purposes, investigating category theory as a framework for theories of physics and biology, for mathematical modelling more generally, and for the structure of scientific theories. Book jacket.
Sheaves in Geometry and Logic Elsevier
This book records my efforts over the past four years to capture in words a description of the form and function of Mathematics, as a background for the Philosophy of Mathematics. My efforts have been encouraged by lec tures that I have given at Heidelberg under the auspices of the Alexander von Humboldt Stiftung, at the University of Chicago, and at the University of Minnesota, the latter under the auspices of the Institute for Mathematics and Its Applications. Jean Benabou has carefully read the entire manuscript and has offered incisive comments. George Glauberman, Car los Kenig, Christopher Mulvey, R. Narasimhan, and Dieter Puppe have provided similar comments on chosen chapters. Fred Linton has pointed out places requiring a more exact choice of wording. Many conversations with George Mackey have given me important insights on the nature of Mathematics. I have had similar help from Alfred Aeppli, John Gray, Jay Goldman, Peter Johnstone, Bill Lawvere, and Roger Lyndon. Over the years, I have profited from discussions of general issues with my colleagues Felix Browder and Melvin Rothenberg. Ideas from Tammo Tom Dieck, Albrecht Dold, Richard Lashof, and Ib Madsen have assisted in my study of geometry. Jerry Bona and B.L. Foster have helped with my examina tion of mechanics. My observations about logic have been subject to con structive scrutiny by Gert Miiller, Marian Boykan Pour-El, Ted Slaman, R. Voreadou, Volker Weispfennig, and Hugh Woodin.
Conceptual Mathematics American Mathematical Soc.
Foundations of Relational Realism presents an intuitive interpretation of quantum mechanics, based on a revised decoherent histories interpretation, structured within a category theoretic topological formalism.
Springer Science & Business Media
Sheaves arose in geometry as coefficients for cohomology and as descriptions of the functions appropriate to various kinds of manifolds. Sheaves also appear in logic as carriers for models of set theory. This text presents topos theory as it has developed from the study of sheaves. Beginning with several examples, it explains the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.
Sheaves in Geometry and Logic Springer Science & Business Media
A graduate-level textbook that presents basic topology from the perspective of category theory. This graduate-level textbook on topology takes a unique approach: it reintroduces basic, point-set topology from a more modern, categorical perspective. Many graduate students are familiar with the ideas of point-set topology and they are ready to learn something new about them. Teaching the subject using category theory—a contemporary branch of mathematics that provides a way to represent abstract concepts—both deepens students' understanding of elementary topology and lays a solid foundation for future work in advanced topics. After presenting the basics of both category theory and topology, the book covers the universal properties of familiar constructions and three main topological properties—connectedness, Hausdorff, and compactness. It presents a fine-grained approach to convergence of sequences and filters; explores categorical limits and colimits, with examples; looks in detail at adjunctions in topology, particularly in mapping spaces; and examines additional adjunctions, presenting ideas from homotopy theory, the fundamental groupoid, and the Seifert van Kampen theorem. End-of-chapter exercises allow students to apply what they have learned. The book expertly guides students of topology through the important transition from undergraduate student with a solid background in analysis or point-set topology to graduate student preparing to work on contemporary problems in mathematics.
Geometry of Vector Sheaves Oxford University Press
The aim of this book is to construct categories of spaces which

contain all the C?-manifolds, but in addition infinitesimal spaces and arbitrary function spaces. To this end, the techniques of Grothendieck toposes (and the logic inherent to them) are explained at a leisurely pace and applied. By discussing topics such as integration, cohomology and vector bundles in the new context, the adequacy of these new spaces for analysis and geometry will be illustrated and the connection to the classical approach to C?-manifolds will be explained.
Topoi Springer Science & Business Media
Third in a three part set, this volume introduces topos theory and the idea of sheaves.
Categorical Foundations Springer
Category Theory has developed rapidly. This book aims to present those ideas and methods which can now be effectively used by Mathe maticians working in a variety of other fields of Mathematical research. This occurs at several levels. On the first level, categories provide a convenient conceptual language, based on the notions of category, functor, natural transformation, contravariance, and functor category. These notions are presented, with appropriate examples, in Chapters I and II. Next comes the fundamental idea of an adjoint pair of functors. This appears in many substantially equivalent forms: That of universal construction, that of direct and inverse limit, and that of pairs offunctors with a natural isomorphism between corresponding sets of arrows. All these forms, with their interrelations, are examined in Chapters III to V. The slogan is "Adjoint functors arise everywhere". Alternatively, the fundamental notion of category theory is that of a monoid -a set with a binary operation of multiplication which is associative and which has a unit; a category itself can be regarded as a sort of general ized monoid. Chapters VI and VII explore this notion and its generaliza tions. Its close connection to pairs of adjoint functors illuminates the ideas of universal algebra and culminates in Beck's theorem characterizing categories of algebras; on the other hand, categories with a monoidal structure (given by a tensor product) lead inter alia to the study of more convenient categories of topological spaces.
Algebraic Geometry Springer Science & Business Media
Publisher Description
Categories for the Working Mathematician Springer Science & Business Media
Sheaves in Geometry and LogicSpringer Science & Business Media
Sheaves in Geometry and Logic Springer Science & Business Media
Higher category theory is generally regarded as technical and forbidding, but part of it is considerably more tractable: the theory of infinity-categories, higher categories in which all higher morphisms are assumed to be invertible. In Higher Topos Theory, Jacob Lurie presents the foundations of this theory, using the language of weak Kan complexes introduced by Boardman and Vogt, and shows how existing theorems in algebraic topology can be reformulated and generalized in the theory's new language. The result is a powerful theory with applications in many areas of mathematics. The book's first five chapters give an exposition of the theory of infinity-categories that emphasizes their role as a generalization of ordinary categories. Many of the fundamental ideas from classical category theory are generalized to the infinity-categorical setting, such as limits and colimits, adjoint functors, ind-objects and pro-objects, locally accessible and presentable categories, Grothendieck fibrations, presheaves, and Yoneda's lemma. A sixth chapter presents an infinity-categorical version of the theory of Grothendieck topoi, introducing the notion of an infinity-topos, an infinity-category that resembles the infinity-category of topological spaces in the sense that it satisfies certain axioms that codify some of the basic principles of algebraic topology. A seventh and final chapter presents applications that illustrate connections between the theory of higher topoi and ideas from classical topology.
An Introduction to Partially Ordered Structures and Sheaves Springer Science & Business Media
This two-volume monograph obtains fundamental notions and results of the standard differential geometry of smooth (CINFINITY) manifolds, without using differential calculus. Here, the sheaf-theoretic character is emphasised. This has theoretical advantages such as greater perspective, clarity and unification, but also practical benefits ranging from elementary particle physics, via gauge theories and theoretical cosmology (`differential spaces'), to non-linear PDEs (generalised functions). Thus, more general applications, which are no longer `smooth' in the classical sense, can be coped with. The treatise might also be construed as a new systematic endeavour to confront the ever-increasing notion that the `world around us is far from being smooth enough'. Audience: This work is intended for postgraduate students and researchers whose work involves differential geometry, global analysis, analysis on manifolds, algebraic topology, sheaf theory, cohomology, functional analysis or abstract harmonic analysis.
Elementary Categories, Elementary Toposes Cambridge University Press
This textbook provides a concise and self-contained introduction to mathematical logic, with a focus on the fundamental topics in first-order logic and model theory. Including examples from several areas of mathematics (algebra, linear algebra and analysis), the book illustrates the relevance and usefulness of logic in the study of these subject areas. The authors start with an exposition of set theory and the axiom of choice as used in everyday mathematics. Proceeding at a gentle pace, they go on to present some of the first important results in model theory, followed by a careful exposition of Gentzen-style natural deduction and a detailed proof of Gödel’s completeness theorem for first-order logic. The book then explores the formal axiom system of Zermelo and Fraenkel before concluding with an extensive list of suggestions for further study. The present volume is primarily aimed at mathematics students who are already familiar with basic analysis, algebra and linear algebra. It contains numerous exercises of varying difficulty and can be used for self-study, though it is ideally suited as a text for a one-semester university course in the second or third year.
Categories for the Working Philosopher Hassell Street Press
Noncommutative geometry, inspired by quantum physics, describes singular spaces by their noncommutative coordinate algebras and metric structures by Dirac-like operators. Such metric geometries are described mathematically by Connes' theory of spectral triples. These lectures, delivered at an EMS Summer School on noncommutative geometry and its applications, provide an overview of spectral triples based on examples. This introduction is aimed at graduate students of both mathematics and theoretical physics. It deals with Dirac operators on spin manifolds, noncommutative tori, Moyal quantization and tangent groupoids, action functionals, and isospectral deformations. The structural framework is the concept of a noncommutative spin geometry; the conditions on spectral triples which determine this concept are developed in detail. The emphasis throughout is on gaining understanding by computing the details of specific examples. The book provides a middle ground between a comprehensive text and a narrowly focused research monograph. It is intended for self-study, enabling the reader to gain access to the essentials of noncommutative geometry. New features since the original course are an expanded bibliography and a survey of more recent examples and applications of spectral triples.
Topology and Geometry Courier Corporation
This truly elementary book on categories introduces retracts, graphs, and adjoints to students and scientists.
Sets, Models and Proofs Cambridge University Press
The first of its kind, this book presents a widely accessible exposition of topos theory, aimed at the philosopher-logician as well as the mathematician. It is suitable for individual study or use in class at the graduate level (it includes 500 exercises). It begins with a fully motivated introduction to category theory itself, moving always from the particular example to the abstract concept. It then introduces the notion of elementary topos, with a wide range of examples and goes on to develop its theory in depth, and to elicit in detail its relationship to Kripke's intuitionistic semantics, models of classical set theory and the conceptual framework of sheaf theory (``localization'' of truth). Of particular interest is a Dedekind-cuts style construction of number systems in topoi, leading to a model of the intuitionistic continuum in which a ``Dedekind-real'' becomes represented as a ``continuously-variable classical real number''. The second edition contains a new chapter, entitled Logical Geometry, which introduces the reader to the theory of geometric morphisms of Grothendieck topoi, and its model-theoretic rendering by Makkai and Reyes. The aim of this chapter is to explain why Deligne's theorem about the existence of points of coherent topoi is equivalent to the classical Completeness theorem for ``geometric'' first-order formulae.